# EVALUATION OF TWO METHODS FOR WELDING DISTORTION SIMULATION

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#### ABSTRACT

For manufacturers of heavy steel constructions, it is a major goal to minimize the number of required prototypes within the design process of welding assemblies. Distortion minimization of complex structures usually needs several tries until the best design, welding parameters and welding sequences can be defined.

ESI GmbH provides two different numerical simulations tools which can predict the distortion of welded structures. One of them is the initially introduced and more general purpose welding simulation package Sysweld<sup>®</sup>. It uses the transient method and thus allows calculations close to full physics on a global model. In this case thermo-mechanical metallurgical effects are coupled, therefore high quality meshes, and long calculation times are required compared to Weld Planner<sup>®</sup>, which is based on the shrinkage model. It requires less effort for meshing and can be computed very quickly. However, simplified models neglect some aspects like phase transformation.

The present work's objective is to figure out which of the two provided simulation tools is more applicable from a practical perspective of industrial heavy steel constructions engineers. Therefore, an exemplary welded structure of a loader crane boom, stiffener plate and console is investigated using both tools comparatively. Simulated distortions are validated against experimental results. Therefore, welding distortions are measured by means of 3D point measurement after welding experiments of the component under consideration. Based on the validated results the preferred method regarding accuracy and required effort for modelling and simulation costs is identified.

Keywords: welding simulation, transient method, shrinkage method, distortion measurement

#### Nomenclature

$\vec{P}_1$	Vector containing coordinates of measuring points before welding experiments	Jx	Joint (x stands for the sequence number)
$\vec{P}_2$	Vector containing coordinates of measuring points after welding experiments	x, y, z	Cartesian coordinates
$\Delta \vec{P}_{1-2}$	Vector containing calculated differences of the coordinates of the measured points	z +	Positive distortion along z-axis
HAZ	Heat affected zone	z —	Negative distortion along z-axis
DOF	Degree of freedom	JMAK	Johnson-Mehl-Avrami-Kolmogoroff Model

#### INTRODUCTION

To this day welding remains one of the most important joining technologies within steel industries. Especially when it comes to manufacturing of heavy and big structures the effort which is put into production and process optimization can be enormous [1], [2]. Often several prototypes and changes in the design are required until the best welding process can be defined. Among further challenges minimization of welding distortion is one of the most commonly tasks which becomes obvious. This is specifically an issue regarding big assemblies comprising a high number of joints since costs for an experimental analysis of different approaches are rising with the amount of material required and effort for carrying out. Years of experience are a decisive factor when it comes to determine the optimum.

Introducing the finite element method back in the 1950s a new technology appeared to hit all fields of engineering. Basically it did, but access to this technology has long been reserved for a selected group because high processing power was inaccessible to most engineers. Since the digital revolution this limitation has disappeared and finite element methods are applied to any field of science and engineering. But there is still a huge variance between scientific and applied use of different software tools. Usually the scientific user group applies simulations of very high quality for fundamental research. The exact description of observation in nature is a main objective. Apart from that there is an industrial user group which main objective is to receive quick results around application related tolerances. Although both user groups use the same method their requirements are completely different.

Appling these considerations on simulation of welding distortion another significant argument can be identified: even if processing power raised within the last years it is still a balancing act to apply simulations on big structures. Very fine meshes and high element quality is required within the area of the heat affected zone (HAZ), whereas coarse mesh is required at large structures to reduce the degree of freedom (DOF) which has a critical impact on the required computation time. There are various methods established to face this challenge like local global approach [3], adaptive mesh techniques [4], [5] or simplified methods for estimateing welding shrinkage [6].

The present work relfects two different simulation approaches to estimate the distortion of a welded structure. First simulations where carried out using the transient method which allows consideration of almost any pysical effect based on a number of different models [7]. A moving heat source model, for instance double ellipsoid heat source [8], is used to create the thermal cycle for each weld. The heat source is modeled based on welding process parameters and is described as a function in space and time. In addition to stability criteria, the maximum permissible time increment is limited by the movement of the heat source by half the length of the molten zone. At each time step the transient temperature field is calculated considering temperature dependant material properties. Finally the mechanical and metallurgical reaction of the structure is calculated by applying a coupling procedure on the thermal results of the transient simulation.The results of transient simulations include stresses, strains and the material properties after welding. Typical

applications are detailed investiagtions on sub assemblies, critical loaded objects and validations. The software used to implement this model is Sysweld® provided by ESI GmbH. Secondly, a mechanical replacement model (shrinkage method) is applied to simulate the welding process based on a non physical approach. The proposed method is based on the elasto-plastic shrinkage of material close to the weld joints. The overall shrinkage in the area of a joint is applied within one time increment. The whole welding process of an assembly is modeled by appliying the shrinkage of each new joint within one single time step. Generally it is possible to apply the shrinkage of all joints whitin one single time step too. There is no heat source model required and material bahaviour is reduced to elastic-plastic properties. The results of simulations based on the shrinkage model include distortions and global stress field. Common applicaionts are preliminary investigations, minimisation of distortion on the basis of different clamping conditions and welding seuquences and as well the simulation of large assemblies. The software used to implement this model is Weld Planner® povided also by ESI GmbH.

A representative sample geometry was designed to ensure the desired industrial context. Therefore an exemplary welded structure of a loader crane boom, stiffener plate and console were implemented. The material used for the sampels was S355 J2G3 wich is a low alloyed steel. Standard material properties provided by the software tools were used.

In order of a consolidated evaluation of the results within this work remarkable distortion at the upper side of the boom is discussed. A comparison of the results from simulation and experiment is made by applying of an intersection plane and the subsequent consideration of the intersection line between the distorted surface and the cutting plane.

Mechanical boundary conditions such as clamping conditions were applied identically for both simulations based on the welding experminent. For the experiment the boom pipe was fixed at the bottom with clamps at the front and rear end. For the simulations all degree of freedom were locked (rigid boundary condition) within the described area. The area is highlighted in fig.3 for the transient simulation and fig.7 for the simulation using the shrinkage model. Thermal boundary conditions including initial temperature and boundary conditions for the heat source model were considered within the transient simulation only. The input data for these boundary and initial conditions were taken from measurements carried out during the experiments.

#### WELDING EXPERIMENT

The welding experiments were carried out on a sample geometry which represents typical components within the manufacture of loader cranes shown below at fig.1. The distortion due to the welding procedure was calculated based on the results of two measurements at designated positions carried out before and after welding procedure. The results of these measurements were given by 3d point coordinates (x, y, z) for each position. Overall 174 measuring points were applied on the top surface of the boom, the plate and the rib.

To ensure clearly recognizable positions for the measuring points a grid was applied mechanically on the top surface of the boom pipe (A), the plate (B) and the rib (C). Additionally, reference points where added at the bottom surface of the boom. These reference points were necessary to detect the point of origin and orientate the coordinate

system properly. Additionally, to the sample geometry, fig. 1 shows the welding sequences (J1, J2, J3, J4) and directions. After the plate was joined to the boom the rib was welded.



Fig. 1 Geometry of the sample for the welding experiments. The number of the joints indicates the welding sequence and the white arrows the welding direction.

The distortion of the structure was calculated within an evaluation routine applying several steps to receive continues surface plots based on the measured point clouds before and after the welding experiment.

First the datasets from the measurement system were organised in a structured way. Based on these clean datasets the transformation at each point was calculated. Therefore, the coordinates of the measuring points before (P<sub>1</sub>) and after (P<sub>2</sub>) welding were subtracted. In the formula below  $\vec{P}_i$  represents a vector containing the coordinates in the sequence shown. The first indices specify if the measurement was applied before or after welding and the second one is needed to identify each measuring point.

$$\Delta \vec{P}_{1-2} = \vec{P}_1 - \vec{P}_2 \tag{1}$$

$$\vec{P}_1 = \left[ x_{1,1}, y_{1,1}, z_{1,1}, x_{1,2}, y_{1,2}, z_{1,2}, \dots, x_{1,i}, y_{1,i}, z_{1,i} \right]^T$$
(2)

$$\vec{P}_2 = \left[x_{2,1}, y_{2,1}, z_{2,1}, x_{2,2}, y_{2,2}, z_{2,2}, \dots, x_{2,i}, y_{2,i}, z_{2,i}\right]^T$$
(3)

Since significant changes were observed especially normal to the upper surface of the crane boom further processing of the data was limited to this direction. Therefore, the minor deviations in the x-y plane were not considered within the evaluation routine. The calculated distortions in z-coordinates were applied at the x-y coordinates of the dataset measured before welding  $[\vec{P}_1]$ .

$$\Delta \vec{P}_{1-2} = \left[ x_{1,1}, y_{1,1}, \Delta z_{1-2}, x_{1,2}, y_{1,2}, \Delta z_{(1-2),i}, \dots, x_{1,i}, y_{1,i}, \Delta z_{(1-2),i} \right]^T$$
(4)

Based on these coordinates a linear interpolation algorithm was applied to obtain continuous transitions each point of the  $\Delta \vec{P}_{1-2}$  dataset. Finally, surface plots were generated to visualise the changes due to the welding procedure. An example of these plots is shown on the left side in fig. 2. Overall six samples were welded within the experiments for comparison of the dispersion of the occurring distortion.

The figure presents the distortion in z direction which occurred at the top surface of the boom. At both ends of the boom positive distortion was observed whereas the area in the middle of the surface shows distortion in negative direction.

By applying a sectional plane at the middle of the surface along the x-axis an intersection line is defined. This graph represents the maximum distortion of the structure in a clear way and was used to compare the results of welding experiments and simulation.

Although constant welding parameters and boundary conditions were applied for all experiments and the general trend is comparable, the detailed distortion of the samples varied.

In Figure 2 the illustration on the right side shows the characteristic shape of four samples which represent the maximum and minimum distortion observed. Furthermore, the mean shape calculated from all six samples welded within the experiments is plotted.



**Fig. 2** Left: Surface plot of the calculated distortion based on the measured data before and after the welding experiment. For further comparison a sectional plane was applied to obtain an intersection line. Right: Intersection lines of four different samples. The dashed line shows the mean shape of all six intersection lines calculated from the measurements of each welding experiment.

#### TRANSIENT THERMO-METALLURGICAL SIMULATION

Based on the geometry designed for the sample production, a three-dimensional model of the structure was created using 3d solid elements. Attention was paid to high element density and element quality around the weld seam and the HAZ. Concurrently it has been considered that the number of elements of the overall model should be minimized to reduce

the computation time. Therefore, in a designated distance to the HAZ a continuous transition to a coarser mesh structure was applied.

Fig. 3 gives an overview of the created mesh for the transient simulation in *Sysweld*. Overall the model included more than 219000 elements. The first detail A on the left side shows the mesh around the joints J3 and J4. Based on micrographs of this area the seam geometry was modelled. Below this detail the area of joints J1 and J2 is shown in detail B. Based on the clamping situation during the welding experiments rigid boundary conditions were applied to the highlighted elements at position (C).

For modelling the energy input a double elliptic heat source model [8] was applied using the database of *Sysweld*<sup>®</sup> [7]. Input parameters for the heat source comprise initial time and end time, welding velocity, estimated values for the geometry of the heat source, energy input, and efficiency. The welding trajectory was defined by selecting individual node points along the corresponding weld seam. For this purpose, an algorithm implemented in *Sysweld*<sup>®</sup> can be used which defines a continuous line along the element edges based on the definition of start, end and direction.



**Fig. 3** Finite element model for the transient simulation in Sysweld® The details show certain areas of the mesh. Detail A: cross section of joints J3 and J4. Detail B cross section of joints J1 and J2. Position C: elements which contain boundary condition (rigid clamping condition).

The applied values for the heat source parameters were estimated from the welding experiments and micrographs of the samples and are represented in the table below

Table 1 Farameters of the double emptic near source appried within the transient simulation								
Joint	<b>t</b> <sub>init</sub>	<b>t</b> <sub>end</sub>	Velocity	Energy	Efficiency	Penetration	Width	Length
[/]	[s]	[s]	[cm/min]	[kW/cm]	[/]	[mm]	[mm]	[mm]
J1	0	95.5	30.4	10.5		5	9	18
J2	107	203.0	30.4	10.5	0.85	5	9	10
J3	213	246.5	35.8	9	0.00	4 E	7	15
J4	254	287.5	33.0	o 9		4.5	1	15

Table 1 Parameters of the double elliptic heat source applied within the transient simulation

The same material (S355 J2G3) which was applied for manufacturing the samples for the welding experiments was used for the transient simulation. The material behaviour was modelled by using the material data supplied from *Sysweld*<sup>®</sup>. The material model takes into consideration six different phases of this low alloyed steel. The thermophysical properties

(density, heat conductivity, specific heat) of the material are phase-, and temperature dependent. Young's modulus is considered temperature dependent whereas yield strength and thermal strains are phase-, and temperature dependent. Ten different phase transformation reactions can be considered by applying JMAK- Model [9] and Koistinen Marburger law [10]. The behaviour of the material properties depending on phase and temperature is shown in figure 4.



Fig. 4 Material properties as a function of temperature and metallurgical phase. The fictive phase is used for simulation of the material deposit during welding.

The material deposit is modelled using the "chewing gum method". An fictive phase is assigned to the weld bead at the start of the simulation. Once the temperature rises in a certain area due to the heat input of the moving heat source the fictive phase transforms to austenite. At this approach the simulation software and the metallurigcal model apply activation of elements automatically.

In a first step the results of the transient simulation were verified by comparing the simulated temperature distribution with results from temperature measurements. Figure 5 shows the temperature profile of one measurement compared with the calculated thermal cycle at the same position (starting point of J1 and J2). Several measurements were carried out to compare thermal cycles at different positions.



**Fig. 5** Comparison of the simulated and measured temperature profile. The position for this temperature measurement was underneath the starting point of the first two joints (J1, J2) which is marked at the picture on the right top of diagram.

Figure 5 shows that temperature peaks due to the joints J1 and J2. The simulation is almost exactly in time with the measured temperature profile. The maximum values reached by simulation and experiment show a difference of about 100°C at the first joint and almost the same maximum temperature at the second peak. These deviations can be attributed to a slightly different position of the measuring point form the evaluated nodal point. The result of the simulated temperature distribution was considered to be sufficiently accurate for further calculation of the mechanical behaviour. No further optimisation of the heat source model was applied.

After the verification of the temperature distribution the mechanical calculation was applied. Based on the results of each time step of the transient temperature field calculation the mechanical reaction of the structure was simulated.



Fig. 6 Calculated distortion after each weldment based on the transient simulation in Sysweld®.

Figure 6 shows on the top the developing distortion field step by step after each weldment. The illustration below shows the boom from top view. Areas of positive and negative distortion and the location of the maximum and minimum distortion are marked. Obviously similar effects are simulated like observed at the welding experiment. The open ends of the boom show positive distortion and the area around the joined plate and rib is distorted in the direction of the negative z-axis.

For final comparisons of the results of simulation an experiment the sectional plane introduced earlier was applied to the deformed structure and is discussed in fig. 9.

#### SHRINKAGE MODEL

In the simplified method volume shrinkage is applied within an area around the weld seam. This area is defined by a parameter which represents the diameter of a virtual tube which is applied alongside the nodes of the welding trajectory. Details of the method applied in *Weld Planner*<sup>®</sup> are not available. At all elements of the structure within the tube the volume shrinkage is applied. With this approach the thermal dilation is substituted by a resulting strain neglecting the history [11]. Fig. 7 shows the virtual tube on the left bottom side.

For the simulation with the shrinkage model a mesh of the structure was created using 2d shell elements. In contrast to the meshing for the transient simulation no particular requirements were considered at the areas around the HAZ. The mesh was created by using automatic mesh functions and barley manual rework what allows a quick set up of the simulation model. Even the weld seam geometry was replaced by the shape of triangles.

Only a few parameters were required to finish the simulation model setup. First the thickness of all components was entered. In a second step the material for the components and joints was defined. After that the diameter of the virtual tubes for each weld seam was entered. After the definition of the welding sequence the model setup was finished.

Table 2 sums up the required parameters for setting up the model based on the shrinkage method. The parameter "Bead Width" in table 2 represents the diameter of the tube representing the weld seam shrinkage area.

Joint	Bead Width	Thickness	Sequence	Material		
[/]	[mm]	[kW/cm]	[/]	[/]		
J1	4.4	5 5	1			
J2	11	5.5	2	0055 1000		
J3	0	4.5	3	S355J2G3		
J4	9	4.5	4			

Table 2 Model	parameters	for the	simulation	with the	shrinkage method

Figure 7 gives an overview of the created model for the simulation with the shrinkage method which includes appx. 60000 finite elements. Detail A on the left side of the illustration shows the area of the Joints J3 and J4. Below an example of the tube representing the weld seam is presented.



**Fig. 7** Finite element model for the simulation using the shrinkage model in *Weld Planner*<sup>®</sup> The details show certain areas of the mesh. Detail A: cross section of joints J3 and J4. Detail B shows elements which contain boundary condition (rigid clamping condition). The last picture on the left side shows an example of the virtual tube which is applied at the joints.

The results of the simulation are presented in figure 8. At the top of the figure the developing distortion field is presented after each joint welded. Similar effects like in the transient simulation and the welding experiments can be observed. The detailed view bellow shows that the areas of positive and negative distortion are divided again. The negative distortion reaches its peak at the area of the joint plate and areas of positive distortion can be observed at the both ends of the tube.

Comparing the absolute values of the distortion it is remarkable that especially the positive distortion reaches higher values compared to the transient simulation.



Fig. 8 Calculated distortion after each weldment based on the simulation in *Weld Planner*<sup>®</sup> using the shrinkage method.

#### COMPARISON OF THE DISTORTION

Once the simulation with the transient model and the shrinkage method have been accomplished the results were compared with the results of the distortion measurements based on the experiment. Therefore, the intersection line of both simulation results and the results of the experiments were put into contrast. The 2d-representation of the distortion submits a clear way of comparison. The intersection lines of the simulations were created the same way as described before at the welding experiment (fig. 2).

Fig. 9 shows the distortion along the same path of all welding experiments and simulations accomplished. The six light grey curves represent the section lines of the six welded samples. Based on these lines an averaged line was calculated which represents the mean shape of the experiments – the scatter must be judged to be remarkable. *Weld Planner*<sup>®</sup> and *Sysweld*<sup>®</sup> simulation results are represented by the dashed and dash-dotted lines. The figure shows that the results from both simulations underestimate the magnitude of the positive distortion at the front and rear end of the boom, while the area of the joint plate achieves good agreement in all cases. Generally, the transient simulations results show almost no significant positive distortion whereas the results from the shrinkage method, especially at the front end of the boom reaches almost two millimetres.

For a better understanding of the results it is mandatory to mention that simulations in *Weld Planner*<sup>®</sup> have been performed several times due to the short calculation time required and adjustments of the model parameters led to an improvement of the results. This procedure was not performed for the *Sysweld*<sup>®</sup> simulation due to a high number of influencing parameters and the high calculation time. The results shown in fig. 9 represent the first shot of the transient simulation without performing any optimization on the model. To clarify the question of the deviation of the experiments a closer look is required. The absolute values at the front end of the boom differ up to three millimetres. This means that the fluctuation of the measured values reaches the dimension of the distortion itself.



Fig. 9 Two-dimensional representation of the distortion obtained by simulation and welding experiments.

Due to the fact the welding parameters and boundary conditions have been set constant for all experiments the reason for the difference of the results has to be a different one. A post processing of the measured data before the welding was done gives an evidence that the starting geometry of the samples differed already before the weldment was done. The bending process of the sheet metal to produce the booms for the samples obviously caused variations of the starting conditions for each sample. The influence of residual stresses related to earlier steps within the production process, limits of the tolerances and spring back effects manifests within these variations of the starting conditions.

# APPLICABILITY IN PRODUCTION PLANNING

Beside the evaluation of the accuracy of the simulations achieved, the effort for modelling was captured qualitatively. Since the application of welding simulation as a tool for process and production planning in steel processing companies requires quick results and high flexibility the effort for modelling and applying changes on existing models is a basic requirement. Furthermore, the simulation should provide the opportunity to apply optimization calculations to figure out the best welding sequence. This requires short simulation times too. To ensure comparable results the modelling and simulation was implemented on the identical hardware.

Table 3 gives an overview of the effort required for geometry discretization, modelling, calculation time, memory requirements and preparation of the results. The effort for various adaptations or variant comparisons was summarized based on the experience gained in this work.

Criteria	Transient simulation	Shrinkage Method	
Discretisation of	Using 3d-solid elements requires high effort	t Based on 2d-shell elements th	
geometry	for meshing critical areas like the HAZ in a	meshing can be applied using	
	proper way. Fine and coarse meshes must	automatic meshing tools	
	be manually combined		
Mesh adaption	If using 3d-solid elements changes usually	If using 2d-shell elements	
	cause additional effort.	changes can be applied easier.	
Interval (time/step)	minimum 0,1935 s (auto adapted)	5 (steps)	
DOF	691938	119696	
Set up welding	Applying welding wizards reduces the	Only a few clicks are required	
parameters	amount of work.		
Calculation time	Generally high calculation times	Generally faster.	
Appx. calculation time	250 h	20 min	
Data storage	Raw data of computation and result files	Raw data of computation and	
requirements	can exceed significant amounts	result files are significant smaller	
File size	120 GB	100 MB	
Optimization of the	Due to high computation times transient	Optimization tool is already	
welding sequence	Method is not recommended	implemented	
Post processing	Effort is strongly connected to the	Effort is strongly connected to the	
	respective application	respective application	

**Table 3** Effort for modelling and simulation from a qualitative point of view

#### CONCLUSION

In this paper, two different methods of welding distortion simulation were compared against the results of measured distortion based on welding experiments. A transient simulation was applied using the software *Sysweld*<sup>®</sup> and the shrinkage method was implemented using the software *Weld Planner*<sup>®</sup>. The effort for modelling and simulating for both methods was shown and evaluation criteria were discussed on a qualitative basis.

The described variance of the starting conditions of the samples reflect the daily challenges of applying simulations on industrial use cases beyond ideal surrounding conditions. Each production step is influenced by upstream processes. Related to the welding processes the variance of the supplied metal sheets, sheet metal cutting, bending process can be examples. Not only the deviation of the geometry, but above all the state of stress and hardening of the material due to forming processes matters.

In addition, it can be deduced from the results that a transient simulation needs excellent models. Those are usually not obtained within one step. Rather, a continuous approach to reality is required until the desired degree of accuracy has been achieved.

Obviously, this fact is in direct contradiction with an application of this method for rapid estimation of distortion on large assemblies and its automated optimization. This is precisely the strength of the shrinkage method. It is therefore consistent that in a first step, the results from the simulation with *Weld Planner*<sup>®</sup> are closer to the measured values. The collected knowledge within this work can be summarised by following list:

- Effort for modelling of simulation is very different. Transient simulation requires excellent model quality to receive good results.
- Both methods basically enable the user to estimate welding distortion within the experimental scatter of results. Obviously different application areas are addressed by *Sysweld*<sup>®</sup> and *Weld Planner*<sup>®</sup>.
- Accuracy of both simulations is similar at the presented stage. Further improvement of the transient model probably enables simulations with higher accuracy but is not practical within an industrial application as a fast estimation tool.

In the context of using welding simulation software as a tool for production planning, the deviation of the geometry of the sample, of the ideal CAD geometry, which has been obtained, must be considered. There is a chance for improving the results of any simulation, regardless of the method chosen. Especially in the application of simulations in production planning, this problem needs to be solved. Manufacturers of simulation software are already moving in that direction and offering tools for the complete mapping of the production chain, from the clamping of the components to the forming and different joining techniques.

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