Flow Pattern and Delta Characteristics in a Dam Reservoir

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Abstract

Floods, with huge amount of water and sediment, play an important role in deposition and distribution of sediment in reservoirs. In this paper, the flow pattern and characteristics of delta in a reservoir are experimentally studied using steady and unsteady flows of water and sediment. The experiments were carried out using triangular and trapezoidal hydrographs. The bed load sediment particles were deposited in the reservoir mouth forming a delta. Investigating the flow pattern revealed that, in spite of the full symmetry in geometry and hydraulic conditions of upstream flow, the flow in the reservoir was asymmetric. It was found that although the characteristics of delta depend on the flow pattern, but the flow pattern may be in turn affected by the sedimentation. Sedimentation lead to formation of unstable flow due to fluctuation in flow direction in the reservoir. The specification of delta including: deviation of delta, its length and shape was also addressed in this paper. An exponential equation was developed to predict the length of delta. **Keywords: Flow Pattern, Sedimentation, Reservoir, Flood Flow, Unsteady Flow**

1. INTRODUCTION

The distribution of sediment in reservoirs as a function of reservoir geometry, inflow hydraulic conditions and sediment characteristics is one of the important issues in designing and managing dam reservoirs. Extensive research on sedimentation and formation of delta carried out in the second half of the 20th century, such as Chang et al. [ⁱ], Sugio [ⁱ], Fan and Morris [ⁱ] and Morris and Fan [ⁱ] among others.^v They studied, qualitatively and quantitatively, the geometric characteristics of the delta formed in different geometric and hydraulic conditions. Shieh et al. [^v] investigated the longitudinal and transverse development of delta in a laboratory model. Kostic and Parker [^v] investigated, experimentallyⁱ and numerically, the effects of density current on the delta characteristics. Lai and Capart [^v, ^v, ⁱ] investigated effects of idensity current and rising reservoir water level on the delta. They also studied effect of channel slope and sediment flow rate on delta morphology over rocky beds. Researchers such as Dewals et al. [^x], Dufresne et al. [^x, ^x, ^x, ^x] and Camnasio et al. [^x, ^x, ⁱ] studied effects of sedimentation on the flow pattern in rectangular reservoirs. These studies showed that under certain geometric conditions, despite the presence of symmetric geometry, an asymmetric flow is created in the reservoir. Laboratory observations of Mamizadeh et al. [^x] showed that the flow and sediment distribution is symmetric in a reservoir with sudden expansion.

Although most of sediments are carried to reservoirs by means of unsteady flows, but majority of earlier researches were carried out under steady flow conditions. Few researchers such as Sediqkia $\begin{bmatrix} x \\ z \end{bmatrix}$ and Heydari $\begin{bmatrix} x \\ z \end{bmatrix}$ investigated specifications of delta under flood conditions. In the present study, the flow pattern and sediment deposition in a dam reservoir are investigated experimentally. The experiments are performed in steady and unsteady conditions while the flow is completely turbulent, and the sediments is transported as bed load. Further details of the laboratory model and experiments are given in the next section.

2. MATERIALS AND METHODS

2.1. EXPERIMENTAL SETUP

The model (Figure 1) consisted of an upstream channel as the river model (with length of 3.0 m, width of 0.3 m and slope of 0.007) followed by reservoir model with length of 5.0 m, initial width of 0.3 m and a terminal width of 2.0 m. The slope of the reservoir bed is 0.75 and angle of the side walls of reservoir θ equals to11.8 degrees (the side slope is 0.21). The bed of the channel and the reservoir is made of smooth steel and the walls are made of glass. For the generation of unsteady flow, a system including: pump, inverter, control & programming device was used. The flow rate was measured by ultrasonic flowmeter with accuracy of 1%, and the data was transferred to the flow generation system. The sediment was injected by a sediment feeder at a distance of 2.0 m from the

reservoir mouth in the upstream channel. The sediment feeder was able to feed sediment particles in steady or unsteady rate into the flow. The bed topography (sedimentation/delta) was recorded by a mechanical point gage with a precision of ± 0.05 mm. The sedimentation and flow pattern were also recorded by a 30 fps camcorder.



Figure 1. The scheme of laboratory model (not to scale)

The flow and sediment discharge were selected in such a way that the sediment moves as a bed load and no deposition occurred in the upstream channel, so that the entire volume of injected sediment was deposited in the reservoir. The critical discharge (minimum flow discharge for initiation of motion of sediment particles) was found to be equal to 10 l/s. Uniform Silica with mean diameter of 0.480 mm was used as non-cohesive particles. The relation between flow and sediment discharges was obtained experimentally as given by equation 1.

$$Q_s = (6 \times 10^{-7}) Q_w^{6.07} \tag{1}$$

Where Q_w is the flow discharge (l/s) and Q_s is sediment discharge (g/s).

2.2. EXPERIMENTS

Experiments were conducted under steady and unsteady flow conditions. Each series of steady and unsteady flow experiments includes several experiments with the same characteristics but different durations. In other words, each experiment was carried out several times with different duration and therefore different amount of sediment transport (M_s). The different durations for the unsteady flow means the implementation of successive identical hydrographs. Details of steady and unsteady flow experiments are given in Tables 1 and 2. In this table S represent the steady flow and U represent the unsteady flow. Unsteady flow experiments were carried out with two types of hydrographs; triangular (Tri.) and trapezoidal (Trap.), as schematically shown in Figure 2. The adjacent numbers to S and U represents the flow rate (l/s). Since the base flow discharge for unsteady flow experiments was equal to the critical discharge (i.e. 10 l/s), therefore no sediment was transported by the base flow.

The dimensionless numbers in Tables 1 and 2 are obtained from the following equations:

$V^{*} = \frac{\left(u_{p}h_{p} - u_{0}h_{0}\right)t_{r}}{h_{p}^{2}}$	
(2)	
$P = \frac{\left(h_p - h_0\right)}{u_p t_r}$	
(3)	
$T^* = \frac{ut}{h}$	(4a- for steady flow)
$T^* = \frac{u_p t_r}{h_p}$	(4b- for unsteady flow)

Where V^* , P and T^* are, respectively, dimensionless volume of the hydrograph, unsteady parameter of hydrograph and time scale. The parameters u, h and t are, respectively, upstream flow velocity, upstream flow depth and time. The subscripts p, 0, and r represent, respectively, the peak value, the base flow and the rising limb of the hydrograph.

Experiment Series	Q _w (l/s)	Q _s (g/s)	Upstream Froude number	Flow deviation	T'*
S-12	12	0.6	0.43	N	_
S-14	14	1.5	0.51	Ν	-
S-16	16	5.0	0.60	Y	8411
S-18	18	10.5	0.71	Y	1237
S-20	20	19.0	0.83	Y	2330

Table 1- Details of steady flow experiments

 Q_w and Q_s are, respectively, water and sediment discharge. T'* is the time scale when the flow starts to fluctuate. Y means the occurrence of flow deviation and N means no flow deviation.

Experiment Series	ms (kg)	Q _p (l/s)	Q ₀ (l/s)	t _d (s)	t _r (s)	P*1000	V*	Flow deviation	T'*
U-Tri. 0.5	0.5	15.3	10.0	360	180	0.088	708.4	Y	81800
U-Tri. 1	1.0	17.5	10.0	360	180	0.125	844.4	Y	7881
U-Tri. 2	2.0	20.0	10.0	360	180	0.150	986.2	Y	6060
U-Tri. 4	4.0	20.0	10.0	720	360	0.075	1972.4	Y	5424
U-Tri. 8	8.0	20.0	10.0	1440	720	0.037	3944.8	Y	9862
U-Trap. 0.5	0.5	15.3	10.0	256	90	0.175	653.3	Ν	-
U-Trap. 1	1.0	17.5	10.0	236	90	0.250	684.9	Y	11625
U-Trap. 2	2.0	20.0	10.0	232	90	0.299	778.0	Y	8876
U-Trap. 4	4.0	20.0	10.0	463	180	0.150	1550.5	Y	6651
U-Trap. 8	8.0	20.0	10.0	926	360	0.075	3101.0	Y	5917

Table 2- Details of unsteady flow experiments

 m_s is the sediment mass transported by one hydrograph, Q_p and Q_0 are, respectively, peak and base flow discharge, t_d and t_r are, respectively, total duration and rising limb duration of the hydrograph. T'* is the time scale at the instant when the flow starts to fluctuate. Y means the occurrence of flow deviation and N means no flow deviation.



Figure 2. Different hydrographs used in unsteady experiments: (a) triangular hydrographs and (b) trapezoidal hydrographs

3. RESULTS

3.1. FLOW PATTERN

Laboratory observations revealed that although sedimentation pattern follows the flow pattern, but flow pattern may be in turn affected by sedimentation. Therefore, in following, the non-sediment (clear water) flow pattern is investigated at first and then the flow pattern after deposition of sediments/formation of delta is discussed.

3.1.1. CLEAR WATER FLOW PATTERN

Several primary experiments were carried out without sediment injection to investigate the flow pattern in the reservoir. Colored strings were floated on the water surface to observe the direction of the flow in the reservoir. The schematic diagram of the flow pattern observed in the laboratory is shown in Figure 3. As this figure shows, with the arrival of the flow in the reservoir, despite the full symmetry in the upstream, the flow is diverted to one of the side walls and, therefore, the asymmetric flow is formed. The formation of asymmetric flow, consequently, leads to formation of a vortex in the reservoir.



Figure 3. Schematic diagram of flow deviation and vortex formation in the reservoir

The direction of the flow and formation of vortex in the reservoir are completely random but stable. In other words, by entering the flow into the reservoir, the flow is randomly diverted to the left or right walls of the reservoir, but the direction of deviated flow did not change during the experiment, which means the flow is stable. It was observed that the turbulence caused by the entry of successive hydrographs in non-sediment flow experiments does not change the direction of deviated flow. A clockwise or counterclockwise vortex is then formed in the reservoir according to the direction of flow.

In the present experiments, the inflow Reynolds number was in the range of 33000 to 66000, which produced an asymmetric flow. This is in accordance with the results of the studies on flow pattern in a channel with converging side walls (Cherdron et al. [x], Durst et al. [x], Sobey [x], Fearn et al. [x], Shapira et al. [x], Chiang et al. [x]). These researchers concluded that the flow in a reservoir is symmetric if approach Reynolds number is low, while it is asymmetric if approach Reynolds number is high. The occurrence of asymmetric flow has been also reported in rectangular shallow reservoirs by other researchers such as Stovin and Saul [x, x], Dewals et al. [10], Dufresne et al. [11, 12, 13] and Camnasio et al. [15].

3.1.2. FLOW PATTERN AFTER SEDIMENT DEPOSITION

As already mentioned, the initial direction of the deviated flow is completely random. Therefore, in order to make the initial conditions for all the experiments identical, the initial flow direction was deliberately diverted to the left side before the beginning of the experiment. The recording of data was started after the stabilization of the flow direction.

In the steady flow experiments, it was observed that by entering the sediment into the reservoir, the depositional pattern of sediments follows the flow pattern and the sediments are initially deposited in the reservoir in the same direction as the main flow (to the left). In the low rates of flow (i.e. $Q_w=12$ and 14 l/s), the flow direction was fixed initially (to the left) during the experiment and no fluctuation in flow direction was observed after deposition of the sediments. Therefore, the flow pattern is stable even after sedimentation in low rates of steady flow. But in higher amounts of steady flow (i.e. $Q_w=16$, 18 and 20 l/s), the deposition of sediments gradually affects the flow pattern and causes the flow not to be stable further. In the experiment S-16 ($Q_w=16$ l/s), sedimentation causes fluctuation in the direction of flow, but the fluctuation gradually decreases with the delta development. In the experiments S-18 and S-20 ($Q_w=18$ and 20 l/s), the flow direction becomes rapidly symmetrical with a slight fluctuation, and then the flow remains to be symmetrical until end of the experiments. The vortex flow in the reservoir disappears when the symmetry flow occurs. It is clear from Table 1 that the possibility of fluctuation in the steady flow depend on the upstream Froude number, although there is no explicit relationship between them. There was no fluctuation in the flow for Froude numbers less than 0.51 (i.e. for experiments S-16, S-18 and S-20.

In the unsteady flow also, the sediment deposition following the flow pattern is initially deviated to the left, but the deposition gradually fluctuates in the flow direction. Figure 4 shows a typical flow deviation in a U-Tri. experiment. The flow, which is marked with red string on the water surface, is diverted to the right at the

beginning of the falling limb of hydrograph. The direction of sediment path has also diverted to the right with a spatial delay relative to the flow.



Figure 4. The flow and sediment diverted to the right in falling limb of the hydrograph (Flow from left to right)

From table 2, it is clear that the dimensionless volume of the hydrograph (V*) is an effective factor influencing fluctuation of the flow direction. The time scale at which the flow starts to fluctuate (T'*) decreases in both triangular and trapezoidal hydrographs by increasing V* (with the exception of the U-Tri. 8 experiment). In U-Trap. 0.5 experiment, with the lowest value of V*, no fluctuation was observed. Also, in U-Tri. 0.5, the flow fluctuation occurs at longer duration as compare to other experiments.

The flow fluctuation was recorded during unsteady flow experiments in 90 seconds intervals, with the sign +1 for flow deviation to the right, 0 for symmetrical flow and -1 for flow deviation to the left. The fluctuation of flow for both types of hydrographs is depicted in Figure 5. It is observed that after the first change in direction of flow, the flow is no longer stable and the flow direction is continuously fluctuating by the entry of successive hydrographs into the reservoir.

The observations revealed that the flow is diverted to the central line of the reservoir, with a short delay after the peak of hydrograph. Then the flow direction may change to the opposite direction in the falling limb of the hydrograph, or return to its original direction again. The former was usually observed in the triangular hydrographs, and the later was usually observed in the trapezoidal hydrographs. Thus, the flat peak of the trapezoidal hydrograph leads to a weak flow fluctuation and further stabilization of flow, as seen in figure 5.





3.2. SEDIMENTATION PATTERN

For low rates of steady flow, the delta direction was always toward the left side (Figure 6a, b). While for higher values of steady flow rates ($Q_w=16$, 18 and 20 l/s), the delta direction, following the flow, changed gradually to symmetrical state. Figure 6c, d, e. shows the symmetry in delta at the end of experiment.

For unsteady flow, the fluctuation of delta direction decreases with development of delta, although the flow continuously fluctuates. This means that the dependency of the delta direction on the flow direction decreases gradually. To describe the sediment pattern in more details, some specifications of delta including the delta deviation parameter, delta length and delta convexity parameter are investigated.

Long-Term Behaviour and Environmentally Friendly Rehabilitation Technologies of Dams (LTBD 2017) DOI:10.3217/978-3-85125-564-5-020



Figure 6. Delta direction in steady flow (Flow from left to right)

3.2.1. DELTA DEVIATION PARAMETER (Ψ)

In order to estimate deviation of the delta from the central line of reservoir, the parameter ψ is introduced, which is equal to the slope of the line connected the delta forehead and the center line of the channel (see Figure 6a). Deviation of the delta is obtained from equation 5, where, Yt is the transverse distance of the delta forehead from the channel center line and X_t is the length of the delta. The value of ψ can be positive, negative or zero, which in turn means the direction of the delta to the right, left or symmetric.

$$\psi = \frac{Y_t}{X_t} \tag{5}$$

The variation of the parameter ψ versus the dimensionless mass of sediment transported into the reservoir (M_s^*) is depicted in Figure 7. The dimensionless mass of the sediment is calculated by equation 6, where M_s is the total mass of sediment transported into the reservoir, ρ_s is the sediment specific mass, g is the gravity acceleration and h is the flow depth. The peak flow depth is used in equation 6 for unsteady flow.

$$M_s^* = \frac{M_s}{\rho_s g h^3} \tag{6}$$

Figure 7 shows that ψ is negative at small values of M_s^* , which means that the delta initially is deviated to the left. For steady flow (Figure 7.a), the parameter ψ remains negative for experiments S-12 and S-14, fluctuates around the horizontal axis for experiment S-16, and gets close to zero for experiments S-18 and S-20. High fluctuations of ψ in experiments with triangular hydrograph (Figures 7.b) are observed, while the fluctuations are reduced significantly for the trapezoidal hydrograph (Figure 7.c). Less fluctuations in trapezoidal hydrograph is due to weak flow fluctuations resulted from the flat peak of the trapezoidal hydrograph. Note that the value of ψ is always negative for U-Trap. 0.5, because the flow direction never changes for this type of hydrograph. The absolute value of delta deviation parameter, when the sediment mass increases and the delta is developed, remains less than 0.21 which is equal to the side wall diversion angle of the reservoir.



Figure 7. Variations of deviation of delta with mass of sediment

3.2.1.1. LENGTH OF DELTA (X_T)

The analysis demonstrated that despite of different sediment patterns formed under different upstream hydraulic conditions, the length of delta is almost identical. The relation between the dimensionless length of delta ($X_t^*=X_t/h$; where h represents the flow depth for steady flow and peak flow depth for unsteady flow) and the dimensionless mass of sediment (M_s^*) is depicted in figure 8 for steady and unsteady flows. It is clear that dimensionless length of delta in all experiments increases exponentially with increasing the dimensionless mass of sediment. The following equation was considered for the dimensionless length of delta:

$$X_t^* = a(M_s^*)^b \tag{7}$$

in which an and b are empirical values obtained by using the experimental data as given in Table 3. The mean absolute prediction error (MAPE) calculated from equation 8 is also included in this table. Here M_i and P_i are, respectively, the measured value and the predicted value, and n represents the number of data.



Figure 8. Length of delta versus mass of sediment

The small values of MAPE given in Table 3 reveals that equation 7 is a reliable to predict the length of delta.

Table 3- The values of a and b and MAPE for Equation 7

Series	а	b	MAPE
S	3.155	0.409	0.09
U-Tri.	3.442	0.362	0.05
U-Trap.	3.159	0.383	0.07

3.2.2. DELTA CONVEXITY PARAMETER (η)

The forehead of the delta is a convex curve while the convexity depends on the upstream flow conditions. The parameter η is defined by equation 9 to estimate the convexity of delta, where X_r and X_l are the length of delta at right and left sides, respectively. The value of this parameter is always between 0 and 1, while bigger values represents further convexity at forehead of delta and vice versa. For the extreme conditions, where the length of delta at the right and left sides are equal to the central length of delta, the forehead of delta is perpendicular to the delta central line. In this case, the convexity parameter is zero. The variations of the parameter η versus the dimensionless sediment mass is plotted in Figure 9.



Figure 9. Variations of the convexity parameter versus sediment mass

It is observed that value of the maximum convexity is about 0.8. Furthermore, irrespective of the fluctuations in the graphs, the delta convexity has a decreasing trend with increasing M_s^* . Decreasing the convexity means faster growth of the left and right sides of delta than its central line. Therefore, the delta develops in the longitudinal direction in the early stages, and then gradually develops in transverse as well as in longitudinal directions. In other words, with development of delta, its enlargement becomes more uniform in a transverse direction leading to decrease of convexity.

The fluctuations in the convexity graphs indicate that the delta is not uniformly developed in the longitudinal direction. In the other words, the delta develops in longitudinal direction in one step and then develops in the transverse direction in the next step, while this steps are alternately repeated.

4. CONCLUSION

In this research, the flow and sedimentation pattern in dam reservoir is investigated under steady and unsteady flows. For this purpose, a laboratory model including two separate systems for generating hydrographs and sediment graphs was used. The results were discussed in two sections of flow pattern and sedimentation pattern. The observations revealed that the symmetric inflow, was randomly diverted to the left or right side of the reservoir and created a clockwise or counterclockwise vortex flow in the reservoir. The deposition of sediments may lead to change of flow pattern by fluctuations in flow direction. The sedimentation in the steady flow leads finally to a symmetrical flow. But in the unsteady flow, at the beginning of the falling limb of the hydrograph, a deviation in direction of flow occurred and, by passing successive hydrographs, the vortex direction changed frequently. This unstable flow pattern causes the delta direction to change with a small temporal and spatial delay relative to the flow. The analysis of the results showed that, unlike other delta specifications, delta length is an exponential function of the volume of sediments transported into the reservoir. A delta convexity parameter was introduced to explain the shape of the delta. It was found that the maximum delta convexity value is equal to 0.8 while it decreases with development of delta, which means growth of sides of the delta is faster than its central line. The delta develops in longitudinal direction and then in the transverse direction alternately.

5. **REFERENCES**

- 1. Chang, H.Y., Simons D.B. and Brooks R.H. (1967), "The Effect of Water Detention Structures On River and Delta Morphology," Symp. On River Morphology, General Assembly Iugo, Bern, Switzerland.
- Sugio, S. (1972), "Laboratory study of degradation and aggradation a discussion," Proc. ASCE, Vol. 98, No. WW4.
- 3. Fan, J. and Morris, G. (1992), "*Reservoir sedimentation. I: delta and density current deposits*," Journal of Hydraulic Engineering, Vol. 118, pp. 354-369.
- 4. Morris, G. and Fan, J. (1998), "Reservoir Sedimentation Handbook, Design And Management Of Dams, Reservoirs, And Watersheds For Sustainable Use," McGraw-Hill companies, Washington. 1800p.
- 5. Shieh, C., Tseng, C., and Hsu, M. (2001), "Effect Of Sediment Gradation On The Geometry Of River Deltas," International Journal of Sediment Research, vol.16, pp. 45-59.
- 6. Kostic, S. and Parker, G. (2003), "Progradational Sand-Mud Deltas in Lakes and Reservoirs, Part 2, Experimental and Numerical Simulation," Journal Of Hydraulic Research, Vol.4, No.2, P.P. 127-140.
- 7. Lai, S. Y. J. and Capart, H. (2007), "Two-Diffusion Description of Hyperpycnal Deltas," J. Geophys. Res., 112, F03005.
- 8. Lai, S. Y. J. and Capart, H. (2008), "*Response of Hyperpychal Deltas to A Steady Rise in Base Level*," 5th IAHR Symposium on River, Coastal and Estuarine Morphodynamics.
- 9. Lai, S. Y. J. and Capart, H. (2009), "*Reservoir Infill by Hyperpycnal Deltas Over Bedrock*," J. Geophys. Res. Lett., 36, L08402.
- 10. Dewals, B.J., Kantoush, S.A., Erpicum, S., Pirotton, M. and Schleiss, A.J. (2008), "*Experimental and numerical analysis of flow instabilities in rectangular shallow basins*," Environ Fluid Mech. 8:31–54.
- 11. Dufresne, M., Vazquez, J., Terfous, A., Ghenaim A., Poulet, J.B. (2009), "*Experimental investigation and CFD modelling of flow, sedimentation, and solids separation in a combined sewer detention tank*," Computers & Fluids, Vol. 38, pp. 1042–1049.
- Dufresne, M., Dewals, B.J., Erpicum, S., Archambeau, P. and Pirotton, M. (2010), '*Experimental investigation of flow pattern and sediment deposition in rectangular shallow reservoirs*," International Journal of Sediment Research, Vol. 25, No. 3, pp. 258–270.
- 13. Dufresne, M., Dewals, B.J., Erpicum, S., Archambeau, P. and Pirotton, M. (2010), "*Classification of flow patterns in rectangular shallow reservoirs*," Journal of Hydraulic Research, Vol. 48, No. 2, pp. 197–204.
- Dufresne, M., Dewals, B.J., Erpicum, S., Archambeau, P. and Pirotton, M. (2011), "Numerical Investigation of Flow Patterns in Rectangular Shallow Reservoirs," Engineering Applications of Computational Fluid Mechanics, Vol. 5, No. 2, pp. 247–258.
- 15. Camnasio, E., Orsi, E. and Schleiss, A.J. (2011), "*Experimental study of velocity fields in rectangular shallow reservoirs*," Journal of Hydraulic Research, Vol. 49, No. 3, pp. 352–358.
- Camnasio, E., Erpicum, S., Orsi, E., Pirotton, M., Schleiss, J. and Dewals, B. (2013), "Coupling between flow and sediment deposition in rectangular shallow reservoirs," Journal of hydraulic research, Vol. 51, No. 5, pp. 535–547.
- 17. Mamizadeh, J., Ayyoubzadeh, S.A. and Banihashemi, M.A. (2012), "*Experimental study of hydraulic-sediment properties on deltaic sedimentation in reservoirs*," International Research Journal of Applied and Basic Sciences, Vol., 3 (4), 810-816.
- 18. Sediqkia, M. (2011), "Experimental study on the Effect of Inflow Hydrograph and Non-uniform Graded Sediment Particles on the Rate of Progression and Delta Pattern in Reservoir," MSc Thesis, Tarbiat Modares University (in Persian).
- 19. Heydari, M. (2013), "Experimental study of the effect of internal flood hydrograph and non-uniform graded sediment particles on the rate of progression and delta pattern," MSc Thesis, Tarbiat Modares University (in Persian).
- 20. Cherdron, W., Durst, F. and Whitelaw, J.H. (1978), "Asymmetric flows and instabilities in symmetric ducts with sudden expansions," Journal of Fluid Mech., 84(1), 13-31.

- 21. Durst, F., Melling A. and Whitelaw J. H. (1974), "Low Reynolds number flow over a plane symmetric sudden expansion," J. Fluid Mech., Vol. 64, part 1, pp. 111-128.
- 22. Sobey, I. J. (1985), "Observation of waves during oscillatory channel flow," J. Fluid Mech. 151,395-426.
- 23. Fearn, R. M., Mullin, T. and Cliffe, K.A. (1990), "Nonlinear flow phenomena in a symmetric sudden expansion," J. Fluid Mech., 211, 595-608.
- 24. Shapira, M., Degani, D. and Weihs D. (1990), "Stability and existence of multiple solutions for viscous flow in suddenly enlarged channels," Computers & fluids, 18, 3,239-258.
- 25. Chiang, T.P., Sheu, W.H. and Wang, S.K. (2000), "Side wall effects on the structure of laminar flow over a plane-symmetric sudden expansion," Computers & Fluids, 29, 467-492.
- 26. Stovin, V. R. and Saul, A. J. (1994), "Sedimentation in Storage Tank Structures," Water Science and Technology, Vol. 29. No. 1-2. pp. 363-372.
- 27. Stovin, V. R. and Saul, A. J. (2000), "Computational Fluid Dynamics and the Design of Sewage Storage Chambers," Water and Environment Journal, Vol. 14, pp.103–110.