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Riemannian Manifold Approach to Scheimpflug Camera Calibration for Embedded Laser-Camera Application

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***Abstract*— This industrial spotlight paper outlines a Riemannian geometry inspired approach to measure geometric quantities in the plane of focus of a Scheimpflug camera in the presence of nonlinear distortions caused by the Scheimpflug model and non-linear lens distortion.**

I. Introduction

For the standard pinhole camera model, the image sensor is parallel to the lens plane and perpendicular to the optical axis. For this type of camera, the points on a plane surface parallel to the lens can be focused sharply on the sensor plane. However, for some specific application scenarios, the surface of interest is oblique to the lens plane. For example, to capture most parts of a tall building facade into the camera view, the camera needs to be tilted upwards with respect to the building facade. In this case, the standard camera is only able to project a narrow line region of the building facade on sharp focus.

It is interesting that the Gaussian focus equation remains valid under the condition that the sensor plane, the lens plane and the object plane intersect in a common line [5].

The Scheimpflug model is encountered in various fields of applications, e.g., architectural photography [6] or in ophthalmology for measuring the thickness of the cornea [3].

In this industrial spotlight paper we address the problem of accurately measuring geometric quantities in the Scheimpflug plane in the presence of non-linear lens distortion effects by following a Riemannian geometry approach [1]. In contrast to state-of-the-art approaches the outlined approach is feasible on embedded platforms and gets along without guessing initial values and iterative optimization steps. Rather, it models the image formation mapping from the Scheimpflug plane to the image plane directly by exploiting point-to-point correspondences and interpolation.

In section II we recall the Scheimpflug model and calibration approaches from literature. Section III-A outlines our parameter-free approach together with experimental results.

II. Scheimpflug Camera

In contrast to the standard pinhole camera, in the Scheimpflug camera model the sensor plane and the lens plane are no longer parallel. See Fig. 1 of a schematic view of the Scheimpflug model. The mathematical model of its image formation mapping can be derived from decomposing the mapping from world coordinates (X,Y,Z) to image pixel coordinates $(\widetilde{x}_t,\widetilde{z}_t)$ into a concatenation of mappings as

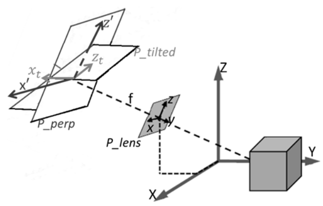

Fig. 1. Scheimpflug camera model: the sensor plane P_{tilt} and lens plane P_{lens} are no longer parallel. The image formation mapping is modeled by means of the virtual parallel plane P_{perp}.

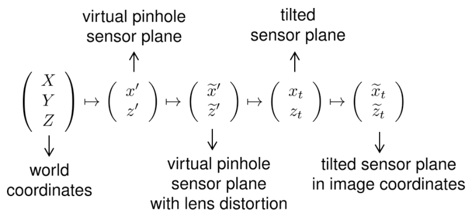

Fig. 2. The process of image formation of the Scheimpflug model according to (1), (2) and (3)

indicated in Fig. 2. First of all, the mapping from (X,Y,Z) to a virtual parallel sensor plane (x',z') models the familiar pinhole camera. By taking non-linear radial and tangential lens distortion effects into account, due to suboptimal shape and mounting of lens, and modeling these effects by means of polynomial functions we obtain

$$\begin{aligned}\begin{pmatrix}\widetilde{x}'\\ \widetilde{z}'\end{pmatrix} := \begin{pmatrix}x'\\ z'\end{pmatrix} &+ \begin{pmatrix}\Delta x\,(k_1\,r^2+k_2\,r^4+k_3\,r^6)\\ \Delta z\,(k_1\,r^2+k_2\,r^4+k_3\,r^6)\end{pmatrix}\\ &+\begin{pmatrix}2t_1\,x'\,z'+t_2\,(r^2+2x'^2)\\ 2t_2\,x'\,z'+t_1\,(r^2+2z'^2)\end{pmatrix},\end{aligned} \tag{1}$$

where $r^2 := \Delta x^2 + \Delta z^2$, $\Delta x := x' - x_0$, $\Delta z := z' - z_0$, (x_0, z_0) are the coordinates of the optical axis on P_{perp}, k_1, k_2, k_3 are radial and t_1, t_2 are tangential distortion parameters. The mapping from $(\widetilde{x}',\widetilde{z}')$ to (x_t, z_t) models the proper Scheimpflug effect by taking the tilt of the sensor plane into account. Let us denote by α the angle between $\widetilde{z}'$ and z_t and by β the angle between $\widetilde{x}'$ and x_t, then due to [4] we obtain

$$\begin{pmatrix}x_t\\ z_t\end{pmatrix} := \lambda\cdot\begin{pmatrix}\widetilde{x}'/\cos\beta+\widetilde{z}'\tan\alpha\tan\beta\\ \widetilde{z}'/\cos\alpha\end{pmatrix} \tag{2}$$

where $\lambda := f/(f-\widetilde{x}'\tan\beta-\widetilde{z}'\frac{\tan\alpha}{\cos\beta})$ and f is the focal length. Finally, we obtain the image pixel coordinates

$$\begin{pmatrix}\widetilde{x}_t\\ \widetilde{z}_t\end{pmatrix} := \begin{pmatrix}S_x & -S_z\cot\theta\\ 0 & S_z/\sin\theta\end{pmatrix}\cdot\begin{pmatrix}x_t\\ z_t\end{pmatrix}+\begin{pmatrix}v_0\\ w_0\end{pmatrix} \tag{3}$$

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where (w,h) denotes the image size in number of pixels, (S_w, S_h) the sensor size in millimeter, (v_0, w_0) the coordinates of the principle point, θ the shearing angle in the sensor coordinate system and $S_x := w/S_w$, $S_z := h/S_h$. To this end we obtain a mapping

$$\Theta : (X,Y,Z) \mapsto (\widetilde{x}_t, \widetilde{z}_t) \tag{4}$$

which depends in total on 17 parameters (6 extrinsic, 2 Scheimpflug angles, 4 intrinsic, 5 distortions coefficients).

III. Scheimpflug Camera Calibration

A standard way for camera calibration in computer vision is the approach of minimizing a functional that measures to which extent the model (4) fits a given set of point-to-point correspondences resulting from a marker positions of a calibration plate. A familiar choice for the functional is the sum of squared projection errors. In particular, the estimate of the extrinsic parameters is not that easily performed. Therefore, usually simplified approximations are used as initial guess. For example, [2] starts from a distortion-free model and derives a first guess of the pinhole camera parameters as an approximation. It is then used as an initialization of a nonlinear bundle adjustment optimization that accounts for distortion and the 2-tilt Scheimpflug angels. In a similar way [4] starts with Zhang's method [7] for estimating the Scheimpflug angels α, β. In a further step, α and β are kept fix and the remaining parameters are estimated, again by using Zhang's method. This procedure is iterated until convergence.

A. Approach for Embedded Laser-Camera Application

The application scenario is about real-time affine reconstruction of geometric quantities by means of an embedded laser-camera system based on a DSP (TMS320DM6435, 700 MHz, 5600MIPS) and a hard-real time requirement of processing a measurement below 10ms. On such a platform the computational effort of trigonometric functions is about 20–40 times higher than standard vector operations. In our approach we exploit the fact that the laser projection plane and the plane of focus of the Scheimpflug camera are congruent. This setting allows a simplification of the general calibration procedure and gets along without the use of computational expensive functions.

Since the mapping (4) reduces to $\widetilde{\Theta} : (X,Z) \mapsto (\widetilde{x}_t, \widetilde{z}_t)$. Instead of solving the inverse problem of identifying the 17 parameters of the Scheimpflug camera model and tackling the problem from a global perspective, we consider the resulting geometric deformation as representation of a Riemannian manifold and exploit its local notions of angle and length of curves for accomplishing measurement tasks. In this view the measurement problem is solved by the following steps: (a) register point-to-point correspondences by means of a sufficiently dense grid of point markers on the plane of focus resulting from straight lines (geodesics in Euclidean geometry) and extraction of the point locations in the image by image processing; (b) determine the neighboring deformed grid points to the sample point; (c) apply 3-spline interpolation for approximate recovery of the corresponding geodesics in the resulting Riemannian manifold; (d) determine the Riemannian coordinates in the local coordinate system given by the geodesics; (e) compute the local inverse in order to obtain the Euclidean coordinates. In contrast to computing the full camera model which involves trigonometric functions and fractions, the outlined approach is also feasible on an embedded system as only polynomials of maximal degree 3 have to be evaluated. Fig. 3 shows an example of a deformed regular grid of calibration points by a Scheimpflug camera and the result of angle measurement based on this approach. The result shows that the systematic angle reconstruction error resulting from non-linear Scheimpflug and lens distortion effects can be reduced substantially which meets the industrial requirements of the specific application.

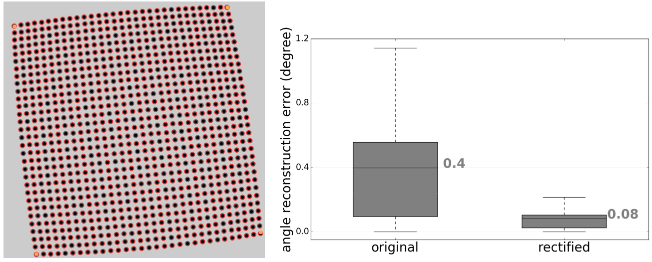

Fig. 3. Left: deformed regular grid of points by Scheimpflug camera and radial and tangential lens distortion: $\alpha = \beta = 5°$, $k_1 = -4.5e^{-3}mm^{-2}$; right: angle reconstruction errors with 249 pairs orthogonal calibration lines and 286 pairs test lines with different inclined angles (left box: original lines with the same distortion as the grid, mean = 0.397°, std = 0.431°; right box: distortion rectified lines, mean = 0.082°, std = 0.084°.)

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